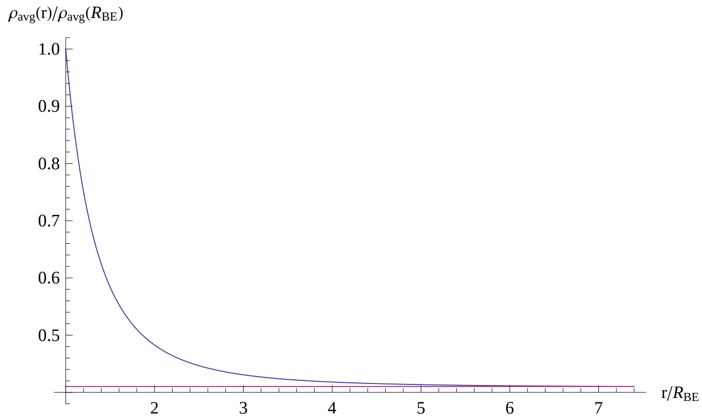
▪ Density at the Bonnor Ebert radius (Rbe) approximation

To begin, let’s define the start of collapse the time when Pram>Pcrit (call it t_c). It seems from the plots that tc $\gtrsim$ 2.3 myr.

A plot for average density as function of radius in units of $\rho(R_{be})$ and R_{BE}, where $\rho_{avg}(R_{BE})$ is the average density of the BE sphere, can give us an idea of how good an approximation homologous collapse is to describe the density at the sphere’s outer edge:



(Note $\rho_{avg}(R_{be}) = 5.77351 \times 10^{-22}$ g / cc and Rbe = 4.99×10^{18} cm). In this plot the horizontal line is $\rho_{avg}(7.4\,\text{Rbe}) / \rho_{avg}(\text{Rbe})$, that is the average density of a sphere with r = 1/2 length of box. This plot shows that approximating the BE sphere as a uniform density sphere is a good approximation only for large radii (r $\gtrsim$4Rbe). For radii below this, a higher density should be used to approximate the average density of the BE sphere as uniform.

The free fall time associated with $\rho_{avg}(7.4\,\text{Rbe})$ is:

$$\text{tff} = \sqrt{\frac{3\pi}{32 * G * \rho_{avg}(7.4\,\text{Rbe})}}$$

$= 1.36543 \times 10^{14}$ s, or ~ 4.3 myr. Note that this is about 2x off of t_c, defined above, indicating that using the average density of the entire box may be inadequete as an approximation for homologous collapse.

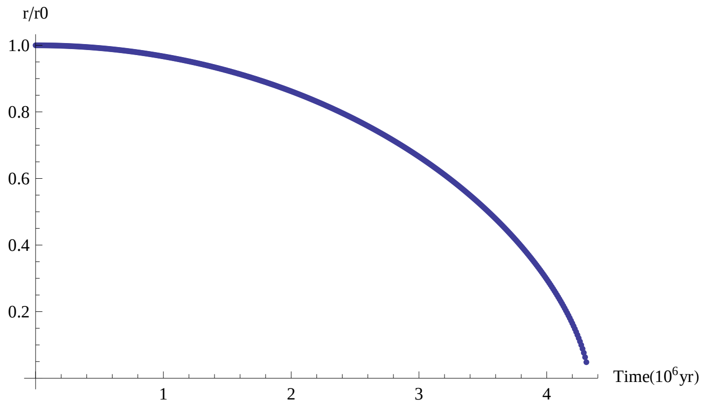
To get a good idea for how the collapse proceeds, one can numerically solve for a given shell’s r(t) given the equation of motion:

$$\xi + \frac{1}{2}\text{Sin}[2\,\xi] == t * \chi, \text{ where } \chi = \sqrt{\frac{8\pi}{3} * G * \rho\text{avg}}\,.$$

By supplying a time to solve for ξ, one can find the radius to which the shell has fallen, r, given its initial radius, r_0 using:

$$\text{Cos}^2\,\xi = \frac{r}{r0}$$

Setting r=Rbe (the radius to which the uniform sphere collapses), one can solve for the initial radius of the sphere r_0. Here is a plot of r/r_0 vs time obtained from numerically solving the above set of equations using r_0=7.4Rbe and $\rho_0 = \rho_{avg}(7.4\,\text{Rbe})$:



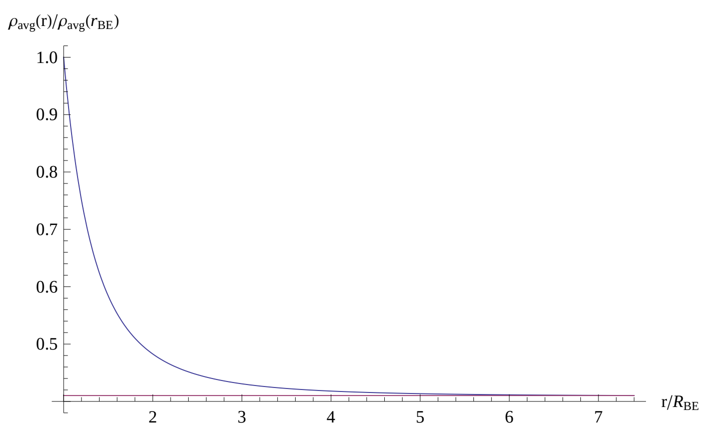
Note that the curve asympotically approaches the free fall time. Now, from my plots above in the results section, we can see that $P_{ram} > P_{crit}$ between 2.3 and 3.07 million years. If we use tc = 2.3 myr in the above system of equations we get r_0 (in units of Rbe) as:

```
ρavg = (Mr * Mbe) / (Vr * Vbe) /. r → 7.4
t = 2.31 * 10^6 * yr;
χ = Sqrt[(8 π)/3 * G * ρavg];
sol = 1 / (Cos[ξ /. First[First[NSolve[ξ + 1/2 Sin[2 ξ] == t * χ, ξ, Reals]]]]^2)
Plot[{(Mr / Vr), ρavg / ρbe}, {r, 1, 7.4},
 AxesLabel → {"r/R_BE", "ρ_avg(r)/ρ_avg(r_BE)"}, PlotRange → All]
Clear[ρavg];
```

2.36842×10^{-22}

1.2307

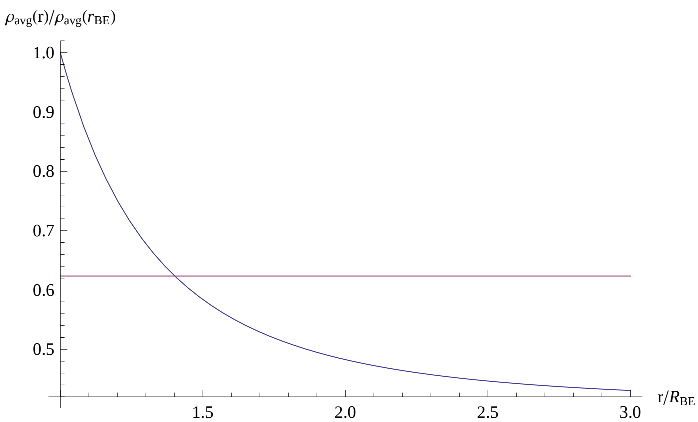
This calculation shows that by using ρ_{avg} of the entire box, the initial radius of the uniform sphere, r_0, to collapse to Rbe within 2.3 myr has r_0= 1.2307 Rbe. However, a glance at r = 1.2 Rbe on the next plot shows that using this ρ_{avg}(7.4 Rbe) is in strong disagreement with the ρ_{avg}(1.2 Rbe):



Thus as is, this is a poor approximation to the initial uniform sphere. Thus, we try to achieve a better match for the densities by increasing the ρ_{avg} used, which changes the r_0 calculated. By doing this iteratively, one can achieve a best fit for the initial density of a uniform sphere of r_0 which matches the actual average density of the sphere and surrounding region. For instance, a $\rho_{avg} = 3.6 * 10^{-22}$works quite well:

```
ρavg = 3.6 * 10^-22;
t = 2.31 * 10^6 * yr;
χ = Sqrt[(8 π)/3 * G * ρavg];
sol = 1 / (Cos[ξ /. First[First[NSolve[ξ + 1/2 Sin[2 ξ] == t * χ, ξ, Reals]]]]^2)
Plot[{(Mr / Vr), ρavg / ρbe}, {r, 1, 3},
 AxesLabel → {"r/R_BE", "ρ_avg(r)/ρ_avg(r_BE)"}, PlotRange → All]
Clear[ρavg];
```

```
1.42237
```



This initial uniform density sphere with radius r_0=1.42Rbe then has $\rho_0 \sim \rho_{avg}$(1.42Rbe) of the corresponding radius of the BE sphere and surrounding medium-- it is a better approximation. Now that we have the parameters of the approximating uniform sphere, namely r_0~1.42Rbe, and $\rho_0 = 3.6 * 10^{-22}\ g/\text{cc}$, and since we know the mass of the shell with radius Rbe< r <1.42Rbe, we can find the density in a thin shell that piled up on the surface of the BE sphere of which we can approximate as having thickness dx=$\mathbf{3 * 10^{17}}$cm:

```
ρfinal = ((Mr * Mbe) - Mbe) / ((4 π / 3) * ((Rbe + dx)^3 - (Rbe)^3)) /. r → sol
```

```
2.31492 × 10^-21
```

This gives a $\boldsymbol{\rho_{approx=}}$ **2.31492×10^{-21} g/cc**.

Ram Pressure Approximation

If we approximate the velocity as $\frac{dr}{dt} = -\sqrt{\frac{8\pi}{3} G\rho_0\, r_0{}^2\left(\frac{r_0}{r} - 1\right)}$, the free fall velocity of a uniform density shell of initial radius r_0 and density ρ_0, at the radius r to which it fell, we can use ρ_0 to get an estimate for the ram pressure at Rbe induced from the homologous collapse:

In[2]:=
```
Vff = -Sqrt[8 π/3 G * ρ0 * r0^2 (r0/r - 1)] /.
  {G → 6.67 * 10^-8, ρ0 → 3.6 * 10^-22, r0 → (1.42) * 4.99 * 10^18 , r → 4.99 * 10^18}
```

$$Vff = -\sqrt{\frac{8\pi}{3} G * \rho 0 * r0^2 \left(\frac{r0}{r} - 1\right)} \;/.\; \{G \to 6.67 * 10^{-8},\ \rho 0 \to 3.6 * 10^{-22},\ r0 \to (1.42) * 4.99 * 10^{18},\ r \to 4.99 * 10^{18}\}$$

Out[2]= -65 130.9

That is, Vff(Rbe) ~ -65,000 cm/s. Now, Pram = $\rho_0\, v_{FF}{}^2$:

In[5]:=
```
Pram = 3.6 * 10^-22 (65 130)^2
```

Out[5]= 1.52709×10^{-12}

Which in computational units is:

In[6]:=
```
ScaledPram = (Pram / Pscale) /. Pscale → 2.765227090397817 * 10^-13
```

Out[6]= 5.52248

Comparing this to Pram plotted above at ~ t=2.3 myr, we see that the estimate is within an order of magnitude.

Lastly, would like to add that by using a ρ_0 = **3.6 * 10^-22** and r_0 = 1.4 Rbe, the asssociated free fall time would be:

In[16]:=

$$tff = \sqrt{\frac{3\pi}{32 * G * 3.6 * 10^{-22}}}$$

Out[16]= 1.10751×10^{14}

or about 3.4 myrs, closer to tc than the previous estimate.